# FORMULA SHEET

**Cartesian and Polar Notation:** $a+jb = re^{j\theta}$ $\quad a = r\cos\theta \quad b = r\sin\theta \quad r = \sqrt{a^2+b^2} \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$

**Complex exponentials & Sinusoids:** $e^{j\theta} = \cos(\theta) + j\sin\theta \quad \cos\theta = \left(\frac{1}{2}\right)(e^{j\theta}+e^{-j\theta}) \quad \sin\theta = \left(\frac{1}{2j}\right)(e^{j\theta}-e^{-j\theta})$

**DT Unit Step:** $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$ **DT Unit Impulse:** $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$

**DT Atomic Decomposition of a Signal:** $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$ **DT Even Signal:** $x[n] = x[-n]$ **DT Odd signal:** $x[n] = -x[-n]$

**DT Basic Signal Operations:** shift: $y[n] = x[n-n_0]$ Flip: $y[n] = x[-n]$ amplitude scaling: $y[n] = \alpha x[n]$

differencer: $y[n] = x[n] - x[n-1]$ accumulator: $y[n] = \sum_{k=-\infty}^{n} x[k]$

**DT Multi-signal Operations:** $y[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \leftarrow$ Linear Combination product: $y[n] = g[n]h[n]$

**DT Linear System:** Suppose the system transformation for $S$ is $x_1[n] \rightarrow y_1[n]$ and $x_2[n] \rightarrow y_2[n]$. $S$ is linear iff $\alpha_1 x_1[n] + \alpha_2 x_2[n] \rightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$ for arbitrary $x_1[n], x_2[n], \alpha_1, \alpha_2$.

**DT Time Invariant:** Suppose the system transformation for system $S$ is $x_1[n] \rightarrow y_1[n]$. $S$ is time-invariant iff $S: x_1[n-n_0] \rightarrow y_1[n-n_0]$ for arbitrary $x_1[n]$ and $n_0$

**CT Unit Step:** $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$ **CT Unit Impulse:** $\delta(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}\{u(t) - u(t-\Delta)\}$

**CT Impulse Decomposition of a Signal:** $x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$ **CT Even Signal:** $x(t) = x(-t)$ **CT Odd Signal:** $x(t) = -x(-t)$

**CT Basic Signal Operations:** shift: $y(t) = x(t-t_0)$ Flip: $y(t) = x(-t)$ amplitude scaling: $y(t) = \alpha x(t)$

differentiator: $y(t) = \frac{d}{dt}x(t)$ integrator: $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

**CT Multi-Signal Operations:** Linear combination: $y(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$ product: $y(t) = g(t)h(t)$

**CT Linear System:** Suppose $S$'s system transformation is $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$. $S$ is linear iff $\alpha_1 x_1[n] + \alpha_2 x_2[n] \rightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$ for arbitrary $x_1(t), x_2(t), \alpha_1, \alpha_2$.

**CT Time-Invariant System:** Suppose the system transformation for system $S$ is $x_1(t) \rightarrow y_1(t)$. The system $S$ is time-invariant iff $x_1(t-t_0) \rightarrow y(t-t_0)$ for arbitrary $x_1(t)$ and $t_0$.

**Finite Sum Formula:** $\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$ $\quad \alpha \neq 1$

**Convolution Sum for LTI Systems:** $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$

**Convolution Integral for LTI systems:** $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = x(t) * h(t)$

**Fourier Transform of Discrete Time Signals:** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ $\quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$

**Response of DT LTI systems to Complex Exponential Signals:** $e^{j\omega_0 n} \rightarrow H(e^{j\omega_0})e^{j\omega_0 n}$ where $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$

**Basic DT Fourier Transform Properties:** $x[n-n_0] \iff e^{-j\omega n_0}X(e^{j\omega})$ $\quad e^{j\omega_0 n}x[n] \iff X(e^{j(\omega-\omega_0)})$

$x^*[n] \iff X^*(e^{-j\omega})$ $\quad x[-n] \iff X(e^{-j\omega})$ $\quad x[n] * h[n] \iff X(e^{j\omega})H(e^{j\omega})$

**Basic DT FT Pairs:** $e^{j\omega_0 n} \iff 2\pi\delta(\omega-\omega_0)$ $\quad \delta[n-n_0] \iff e^{j\omega n_0}$ $\quad u[n] - u[n-N] \iff \frac{\sin(\omega N/2)}{\sin(\omega/2)}e^{-j\omega(N-1)/2}$

**Response of CT LTI Systems to Complex Exponentials:** If $S$ is an LTI system w/ impulse response $h(t)$, then

$e^{j\omega_0 t} \rightarrow H(j\omega_0)e^{j\omega_0 t}$ where $H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt$

**Fourier Transform of CT Signals:** $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $\quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,dt$

**Basic CT FT Pairs:** $\delta(t-t_0) \iff e^{-j\omega t_0}$ $\quad e^{j\omega_0 t} \iff 2\pi\delta(\omega-\omega_0)$ $\quad u(t+T_1) - u(t-T_1) \iff \frac{2\sin(\omega T_1)}{\omega}$

**CT FT Properties:** $x(t) \iff X(j\omega)$ $\quad h(t) \iff H(j\omega)$ $\quad \alpha x(t) + \beta h(t) \iff \alpha X(j\omega) + \beta H(j\omega)$

$x(t-t_0) \iff e^{-j\omega t_0}X(j\omega)$ $\quad x(-t) \iff X(-j\omega)$ $\quad x^*(t) \iff X^*(-j\omega)$ $\quad x(t) * h(t) \iff X(j\omega)H(j\omega)$

$\sin(\omega) = 0$ when $\omega = k\pi$

$\cos = 0$ when $\omega = \frac{(2k+1)\pi}{2}$